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# RESEARCH MEMORANDUM

## PROPELLER LIFT AND THRUST DISTRIBUTION FROM WAKE SURVEYS OF STAGNATION CONDITIONS


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RESEARCH MEMORANDUM

# PROPELLER LIFT AND THRUST DISTRIBUTION FROM WAKE SURVEYS OF STAGNATION CONDITIONS

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## SUMMARY

The Bernoulli equation for nonsteady flow was used to derive formulas for propeller lift and thrust distribution in terms of wake-survey measurements of stagnation pressure rise through the propeller. Lift distributions and over-all values of thrust from wake survey were compared with direct measurements of these quantities at stream Mach numbers up to 0.65.

## INTRODUCTION

In reference 1, Lock and Yeatman derive the formula for propeller thrust in terms of wake-survey measurements. Their formula is the same as is obtained from momentum considerations, but the method of derivation is believed to give greater insight into the mechanics of the wake survey. The analysis of Lock and Yeatman, although complicated at the start by the introduction of the velocity potential, develops the thrust formula in a logical manner for a finite number of blades and periodic flow. From this development it appears that thrust distribution is not the only thing which the wake survey is capable of measuring. For instance, it can be deduced that lift distribution is given more naturally by the wake survey than thrust. Reference 1 shows that the derivation of the thrust formula actually involves neglecting the induced angle of attack; in fact, once this assumption is made, the wake survey can just as reasonably be used to measure torque as thrust.

In this paper it is shown that the wake-survey formulas may be modified for compressible flow without very complicating changes in form. As in reference 1, the total-pressure survey tube is recognized as measuring the time rate of change of the velocity potential. If advantage is taken of the fact that pressure changes are small in the flow about a propeller, the wake-survey formulas for compressible flow remain simple in form.

## SYMBOLS

| | |
|---|---|
| $B$ | number of blades |
| $b$ | blade chord |
| $C_P$ | power coefficient $\left(\dfrac{P}{\rho_1 n^3 D^5}\right)$ |
| $C_T$ | thrust coefficient $\left(\dfrac{T}{\rho_1 n^2 D^4}\right)$ |
| $c_l$ | section lift coefficient |
| $c_n$ | section normal-force coefficient |
| $c_p$ | section power coefficient $(dC_P/dx)$ |
| $c_t$ | section thrust coefficient $(dC_T/dx)$ |
| $D$ | propeller diameter |

$$E = -\int p \, d\left(\frac{1}{\rho}\right)$$

$$h = \frac{\Delta p_t}{p_{t1}} \text{ (See equation (9).)}$$

$$m = \frac{\gamma - 1}{\gamma} \text{ (See equation (9).)}$$

| | |
|---|---|
| $J$ | advance ratio $(V/nD)$ |
| $M_o$ | advance Mach number |
| $M_x$ | helical section Mach number $\left(M\sqrt{1 + \left(\frac{\pi x}{J}\right)^2}\right)$ |
| $n$ | rotational speed, revolutions in unit time |

| | |
|---|---|
| $p$ | static pressure |
| $p_t$ | stagnation pressure |
| $r$ | radius from the rotational axis to a point in the flow |
| $T$ | thrust |
| $t$ | time |
| $V$ | advance velocity. |
| $W$ | resultant velocity of a propeller section |
| $W_o$ | velocity vector (see fig. 1) |
| $w$ | fluid velocity |
| $x$ | radius divided by propeller radius $\left(\frac{r}{(D/2)}\right)$ |
| $\Gamma$ | circulation |
| $\rho$ | mass density |
| $\rho_t$ | stagnation density |
| $\phi$ | velocity potential |
| $\omega$ | rotational speed, radians in unit time |
| $\tau$ | time between blades passing a survey tube $(2\pi/B\omega)$ |
| $\gamma$ | ratio of specific heats |
| $\Delta p_t$ | stagnation pressure rise through propeller |

Subscripts:

| | |
|---|---|
| 1 | denotes free stream condition far ahead of propeller or at some place where flow is practically steady |
| $\{\ \}$ | curly brackets $\{\ \}$ are used to show that a time average is taken |

## DERIVATIONS

The flow field about a propeller, as for a wing, is irrotational with the exception of the thin surfaces of rotational flow which trail from the blades as a result of viscosity. When the effect of the blade wakes on a survey tube is neglected by assuming the flow to be all irrotational, the velocity, the velocity potential, and the state functions of the gas must satisfy:

$$\frac{\partial \phi}{\partial t} = \frac{p}{\rho} + \frac{1}{2} w^2 + E - F(t) \tag{1}$$

where

$$E = -\int p \, d(1/\rho) \tag{2}$$

and the coordinate system is fixed to the survey rake.

Observe that for the propeller wake survey, unlike the wing, the irrotational flow outside the vortex sheets is surveyed and the measurable forces are those arising primarily from circulation rather than viscosity.

Equation (1) is applicable to the compressible flow past a propeller with no restriction on the magnitude of the velocities except that the velocity potential $\phi$ must exist. This equation appears on page 20 of reference 2.

For a compressible flow the following relation still holds:

$$\Gamma = \frac{1}{2} c_l b W \tag{3}$$

where $\Gamma$ is the circulation at a particular radius and is equal to the jump in $\phi$ across a trailing vortex sheet at that radius in the wake.

The relation between pressure and density is assumed to be as follows:

$$\frac{p}{\rho^\gamma} = \text{Constant} \tag{4}$$

which is known to be a good assumption for practical problems in the aerodynamics of compressible flows.

In compressible flow the change in velocity potential between the departure and arrival of a vortex sheet at a total-pressure tube of the survey rake (see fig. 1) is still equal to the jump across a vortex sheet which is equal to the circulation, all at a particular radius. Therefore, the reasoning of reference 1 with equations (1), (3), and (4) gives for conditions at a total-pressure tube behind the propeller

$$\frac{\Gamma}{\tau} = \left\{\frac{p_t}{\rho_t} + E\right\} \tag{5}$$

whereas, far upstream,

$$0 = \frac{p_{t1}}{\rho_{t1}} + E_1 \tag{6}$$

and $F(t) = 0$ because it is zero far ahead where the flow is steady.

Equation (4) substituted in equation (2) gives

$$E = \frac{1}{\gamma - 1}\left(\frac{p}{\rho} - \frac{p_o}{\rho_o}\right)$$

where $p_o$, $\rho_o$ represent some initial state. Subtracting equation (6) from equation (5) and using the expression for $E$ gives

$$\frac{\Gamma}{\tau} = \frac{\gamma}{\gamma - 1}\left(\left\{\frac{p_t}{\rho_t}\right\} - \frac{p_{t1}}{\rho_{t1}}\right) \tag{7}$$

The stagnation density $\rho_t$ behind the propeller is replaced by using equation (4) to give

$$\frac{1}{\rho_t} = \frac{1}{\rho_{t1}}\left(\frac{p_{t1}}{p_t}\right)^{1/\gamma}$$

When this equation is substituted in equation (7), the latter takes the form:

$$\frac{\Gamma}{\tau} = \frac{\gamma}{\gamma - 1}\,\frac{{p_{t1}}^{1/\gamma}}{\rho_{t1}}\left[\left\{{p_t}^{\frac{\gamma-1}{\gamma}}\right\} - {p_{t1}}^{\frac{\gamma-1}{\gamma}}\right] \tag{8}$$

Although equation (8) gives the circulation in terms of total-pressure measurements alone, it is desirable to be able to use the reading of an ordinary liquid manometer which is usually assumed to give $\left\{p_t\right\}$, rather than the averaged quantity in equation (8). A binomial expansion of $p_t^{\frac{\gamma-1}{\gamma}}$ will avoid the difficulty. Only one term of the series need be retained in order to be in keeping with the error in the experimental measurement of $p_t$. Retaining this term leads to a formula which permits easy computation of the lift from total-pressure measurements with a liquid manometer.

Let:

$$\left.\begin{aligned} p_t &= p_{t1} + \Delta p_t \\ h &= \frac{\Delta p_t}{p_{t1}} \\ m &= \frac{\gamma - 1}{\gamma} \end{aligned}\right\} \tag{9}$$

then equation (8) becomes

$$\frac{\Gamma}{\tau} = \frac{1}{m}\frac{p_{t1}}{\rho_{t1}}\left[\left\{(1 + h)^m\right\} - 1\right]$$

and with the expansion of $(1 + h)^m$ this becomes the alternating series

$$\frac{\Gamma}{\tau} = \frac{1}{m}\frac{p_{t1}}{\rho_{t1}}\left[m\left\{h\right\} + \frac{m(m-1)}{2!}\left\{h^2\right\} + \ldots\right] \tag{10}$$

In trying to see the relative magnitude of the terms, it is helpful to make the exponent in the second term of equation (10) appear on the outside rather than the inside of the curly brackets. For this purpose, some assumption is necessary as to the form of h.

Measurements with instantaneous total-pressure measuring devices indicate that the total-pressure variation with time between vortex sheets is approximately parabolic. A typical measurement of this kind is shown in figure 2(a). The pertinent parts of the record are shown in figure 2(b). As would be expected, the total-pressure increment $\Delta p_t$ apparently never comes down to zero at the midpoints. However, a conservative simplifying assumption is that this does happen as shown in figure 2(c), that is, that the origin of the parabola is at $\Delta p_t = 0$. With the assumption that $h$ varies in time as represented in figure 2(c), the quantities $\{h\}^2$ and $\{h^2\}$ are no longer independent, in fact

$$\{h^2\} = \frac{144}{80}\{h\}^2$$

as some simple integrations will show. Equation (10) then becomes

$$\frac{\Gamma}{\tau} = \frac{1}{m}\frac{p_{t1}}{\rho_{t1}}\left[m\ \{h\} + \frac{m(m-1)}{2!}\ \frac{144}{80}\{h\}^2 + \ .\ .\ .\right] \tag{11}$$

For the values

$$m = \frac{\gamma - 1}{\gamma} = 0.286$$

$$p_{t1} = 2000 \text{ pounds per square foot}$$

$$\{\Delta p_t\} = 20 \text{ pounds per square foot}$$

$$\{h\} = 0.01$$

which give a conservatively heavy disk loading, equation (11) becomes

$$\frac{\Gamma}{\tau} = \frac{1}{m}\frac{p_{t1}}{\rho_{t1}}\left[0.00286 - 0.000018 + \ .\ .\ .\right]$$

Therefore, the error when retaining only the first term in equation (11) is less than

$$\frac{0.000018}{0.00286} = 0.0063 \text{ or } 0.6 \text{ percent}$$

It follows that, to a good approximation,

$$\frac{\Gamma}{\tau} = \frac{p_{t1}}{\rho_{t1}}\{h\}$$

[redacted]

or

$$\frac{\Gamma}{\tau} = \frac{\{\Delta p_t\}}{\rho_{t1}} \tag{12}$$

Equation (12) gives the circulation distribution from easily measured quantities. The bracketed term is obtained directly from a liquid manometer, whereas $\rho_{t1}$ is the stagnation density in the undisturbed stream.

Lift coefficient.- The lift distribution is easily obtained from equation (12). The time for one blade to succeed the preceding one is the same as the time between passages of the vortex sheets at a total-pressure tube; therefore,

$$\tau = \frac{2\pi}{B\omega} \tag{13}$$

Then equations (3) and (13) substituted in equation (12) give

$$\frac{\frac{1}{2}c_l\ bWB\omega}{2\pi} = \frac{\{\Delta p_t\}}{\rho_{t1}}$$

with which the lift coefficient may be calculated from stagnation measurements.

With the excellent approximation that $W = W_o$ (see fig. 1), the last equation may be solved for $c_l$.

$$c_l = \frac{J}{B(b/D)\sqrt{1 + \left(\frac{\pi x}{J}\right)^2}} \frac{\{\Delta p_t\}}{\frac{1}{2}\rho_{t1}\ V^2} \tag{14}$$

which is the formula with which $c_l$ was calculated in figure 3. This formula will give slightly low values of $c_l$ depending upon the extent to which profile drag influences the readings of the total-pressure tubes. Because the radical expresses the section velocity at the blade, $x$ must be at the blade with no correction factor concerning the slip-stream contraction.

Thrust.- By projection of the lift in the forward direction, the thrust may be found from equation (14). The induced angle $\alpha_i$ (see fig. 1) and the drag are neglected. The formula for the local thrust coefficient in compressible flow is then

$$c_t = \frac{1}{4} J^2 \pi x \frac{\{\Delta p_t\}}{\frac{1}{2}\rho_{t1} V^2} \tag{15}$$

which is the same as the formula at the bottom of page 6 in reference 3.

Integration of equation (15) gives the over-all thrust coefficient $C_T$ shown in figure 4.

For incompressible flow, equation (15) reduces to the well-known formula for thrust

$$\frac{dT}{dr} = \pi x D\{\Delta p_t\} \qquad < 230 \tag{16}$$

57

which is obtained from the momentum theorem for incompressible flow. The substitutions

$$dT = \rho_1 n^2 D^4 dC_T$$

and

$$dr = \frac{1}{2} D\ dx$$

give equation (16) in notation comparable with equation (15):

$$c_t = \frac{1}{4} J^2 \frac{\pi x\{\Delta p_t\}}{\frac{1}{2}\rho_1 V^2} \tag{17}$$

Equation (17) is for incompressible flow. Equations (15) and (17) are the same for incompressible flow because, in this case, the densities $\rho_{t1}$ and $\rho_1$ are equal and all other terms are the same. Thus, the only change needed to make the established thrust formula (17) apply to compressible flow is the substitution of stagnation density $\rho_{t1}$ for stream density $\rho_1$.

Formula (16) is known to neglect the rotation of the slipstream when derived from momentum thoery. Formula (16) neglects the slipstream rotation and formula (15) neglects the induced angle of attack.

## COMPARISON OF WAKE-SURVEY RESULTS WITH PRESSURE AND OVER-ALL FORCE MEASUREMENTS

An experimental check of the wake-survey formula (14) for lift coefficient is possible. Propeller tests in which normal-force coefficients were obtained at various stations on a 10-foot-diameter propeller have been made in the Langley 16-foot high-speed tunnel. The normal-force coefficients are computed from direct measurements of static-pressure distribution on sections of the rotating propeller. Wake surveys were also made during the same tests.

Lift coefficients from wake-survey measurements and formula (14) are compared with normal-force coefficients from direct measurements of static pressures in figure 3. The tests from which the data were taken are reported in reference 4. In figure 3, solid lines are faired through points plotted from wake-survey data. Plotted points with crosses through them denote normal-force coefficients obtained from direct static-pressure measurements on the blades. The points with crosses should fall on the faired lines except for the small difference between $c_l$ and $c_n$. The dashed lines are faired in through radial stations where a given sectional Mach number prevails. Disagreement is seen to come in around $M_x = 1.0$.

In figure 4, thrust from wake-survey formula (15) is compared with direct thrust measurements. The comparison is not as thorough as that for the lift coefficient because only the integrated value of the wake-survey thrust is shown. The tests from which the data were taken are reported in reference 5. The agreement is fair up to the highest advance Mach number tested.

## CONCLUDING REMARKS

Wake-survey formulas were presented which are applicable to compressible flow and convenient for computation if a slight degree of approximation is accepted. The usefulness of the wake survey has always been in the ease with which force distribution may be obtained from easily measured quantities. Therefore, a small sacrifice in accuracy was considered worthwhile in view of the resulting simplification of computations.

As long as a velocity potential exists no Mach number restrictions are required. However, if for some reason the flow becomes seriously rotational, the formulas presented are in error by some indefinite amount.

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Langley Field, Va.

## REFERENCES

1. Lock, C. N. H., and Yeatman, D. M.: Theoretical Curves Illustrating the Periodic Flow behind an Airscrew. R. & M. No. 1483, British A.R.C., 1933.

2. Lamb, Horace: Hydrodynamics. Reprint of sixth ed. (first American ed.), Dover Publications (New York), 1945.

3. Vogeley, A. W.: Flight Measurements of Compressibility Effects on a Three-Blade Thin Clark Y Propeller Operating at Constant Advance-Diameter Ratio and Blade Angle. NACA ACR 3G12, 1943.

4. Johnson, Peter J.: Pressure Distributions on the Blade Sections of the NACA 10-(3)(090)-03 Propeller under Operating Conditions. NACA RM L50A26, 1950.

5. Evans, Albert J.: Propeller Section Aerodynamic Characteristics as Determined by Measuring the Section Surface Pressures on an NACA 10-(3)(08)-03 Propeller under Operating Conditions. NACA RM L50H03, 1950.

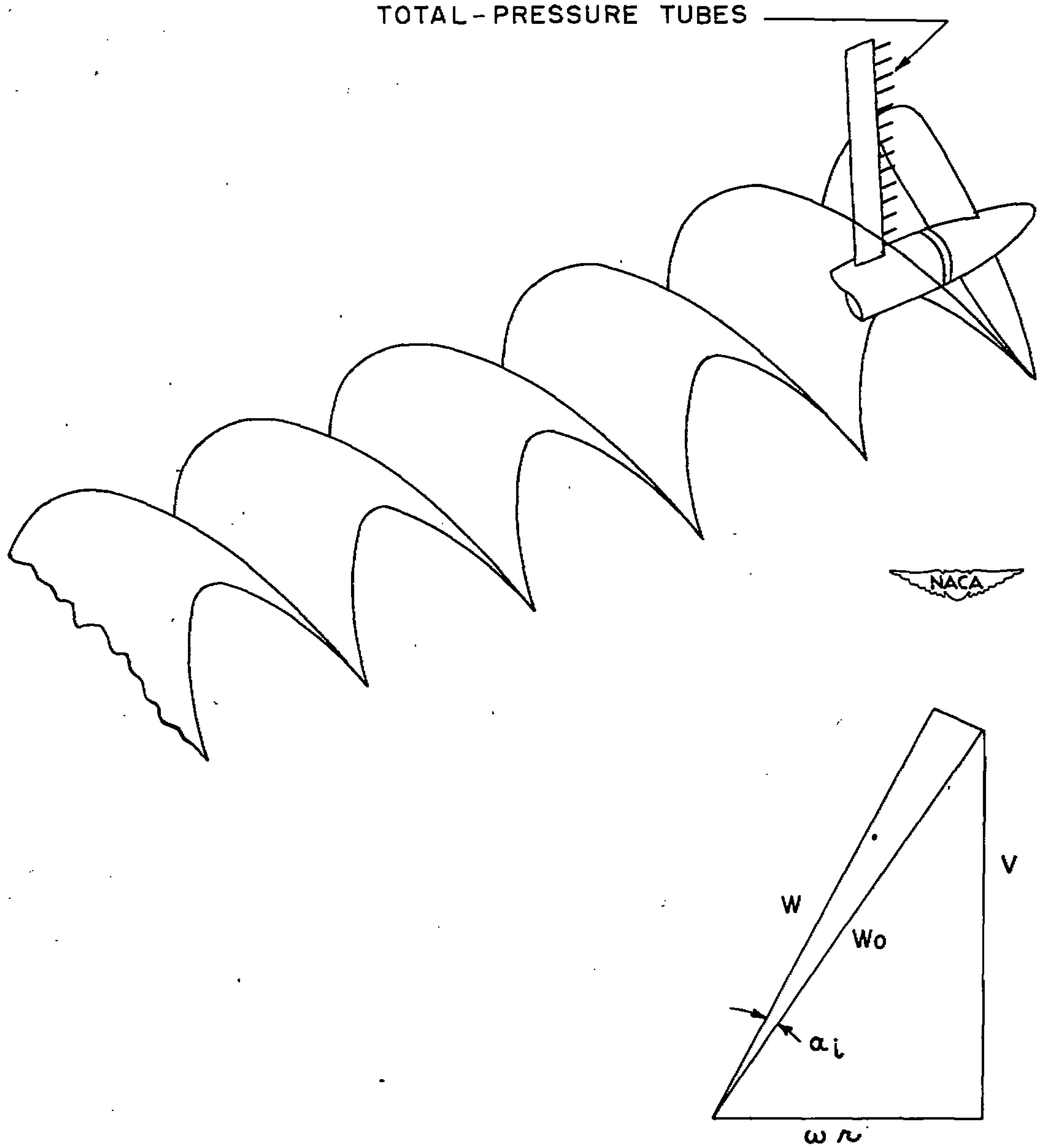



Figure 1.- Schematic diagram of propeller wake and survey setup, with diagram of velocities at the blade.

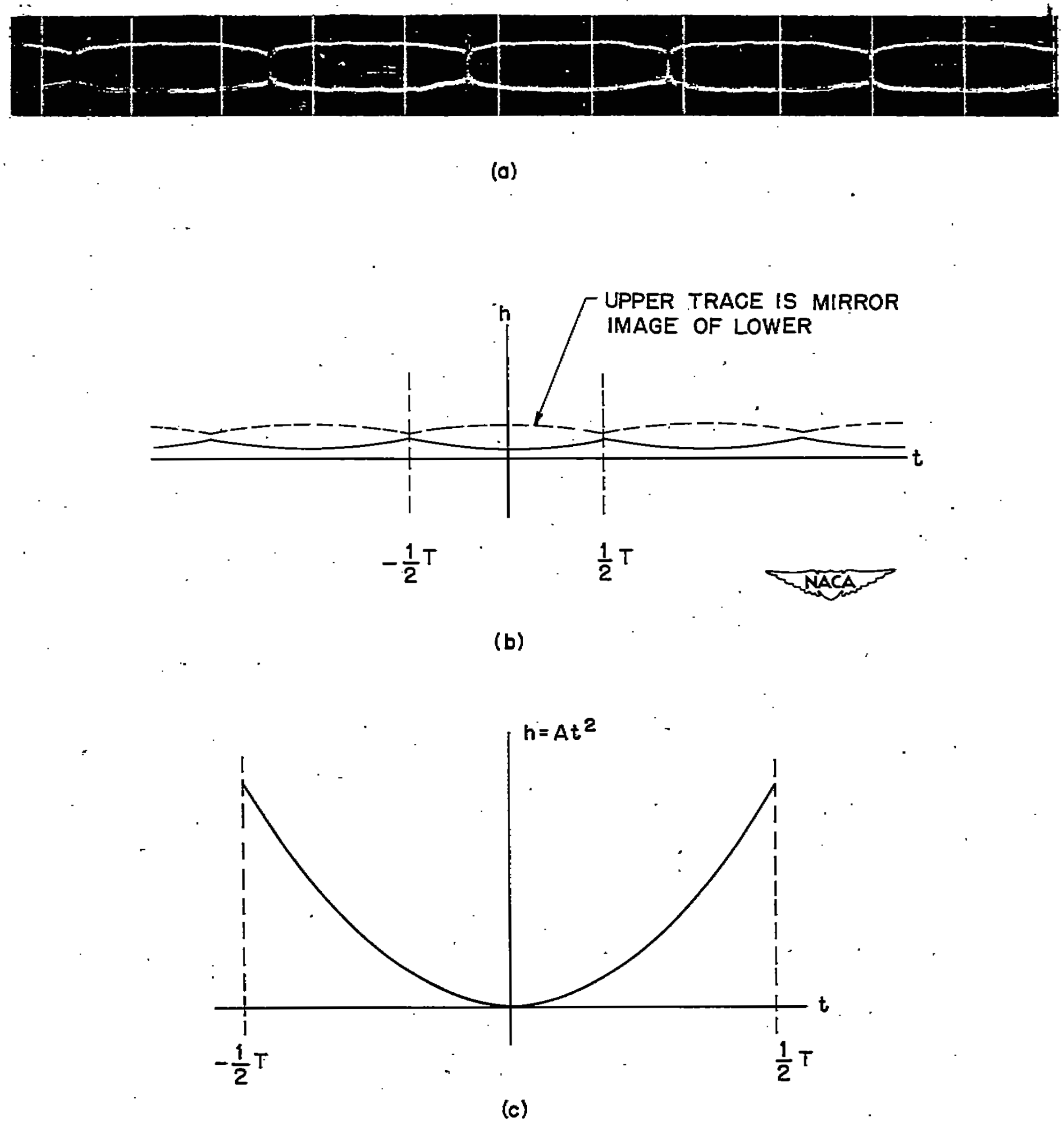
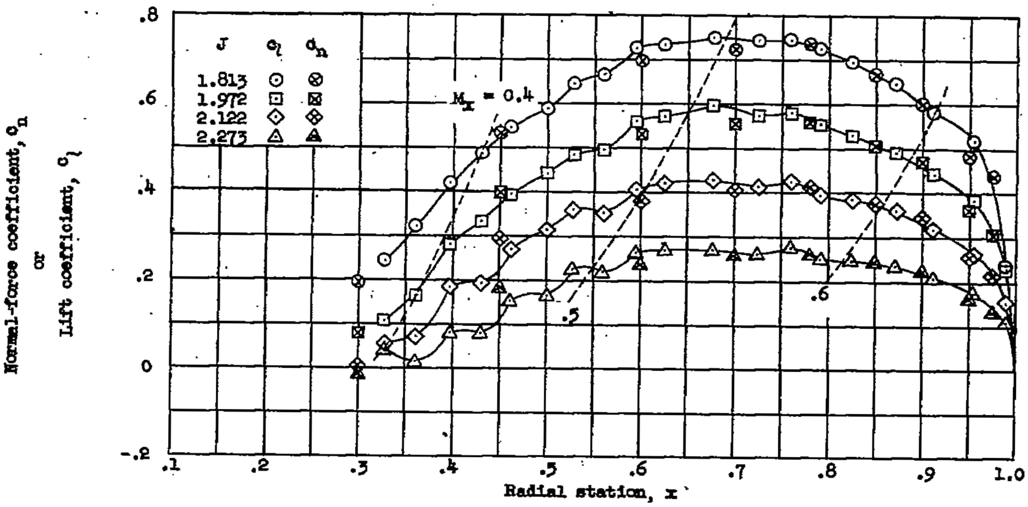
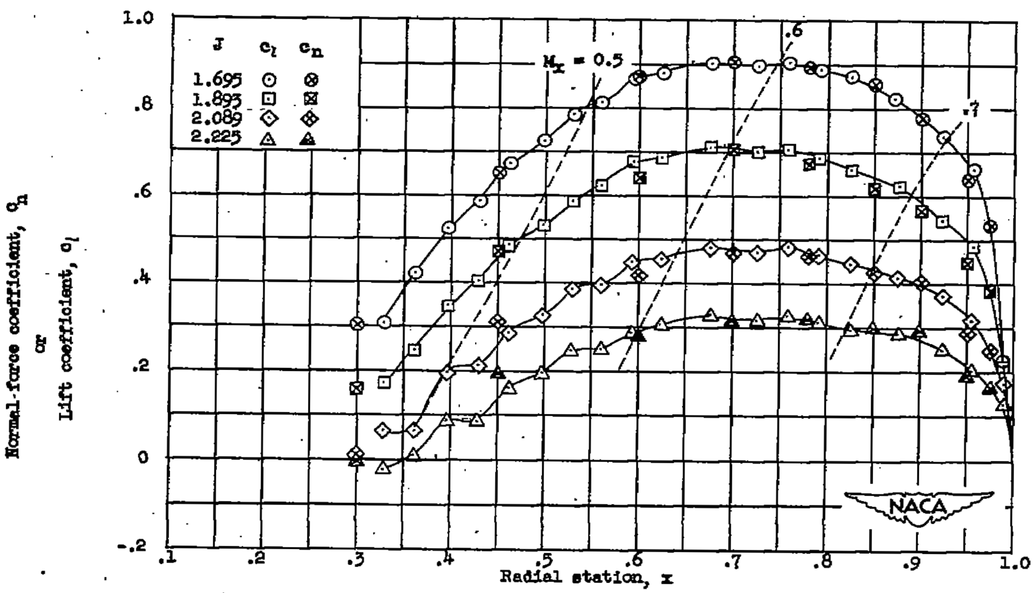



Figure 2.- Actual and assumed time history of $h = \frac{\Delta p_t}{p_{t1}}$.

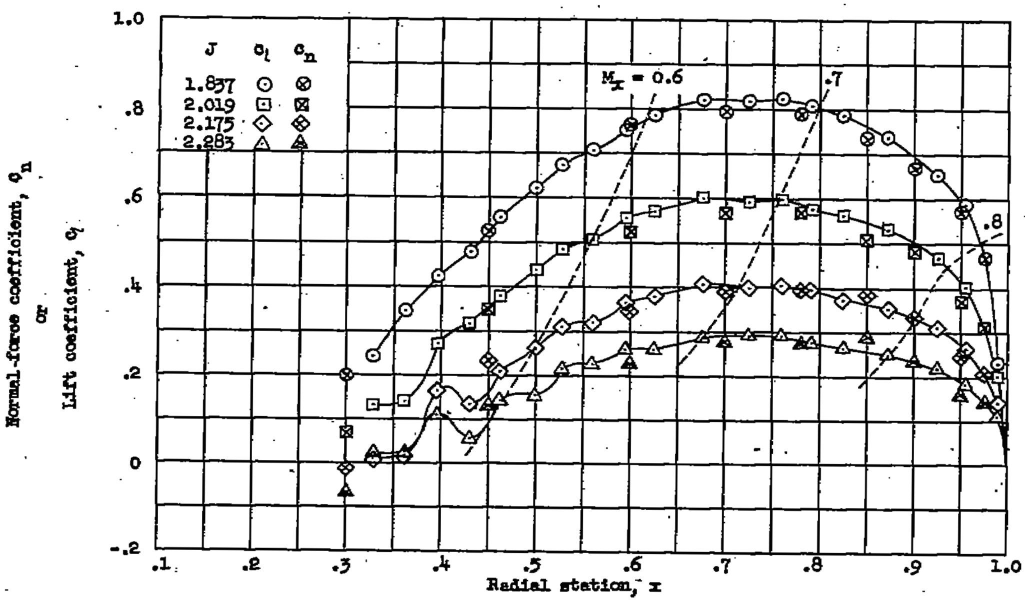



(a) 1140 rpm; $\beta_{0.75R} = 45^{\circ}$.

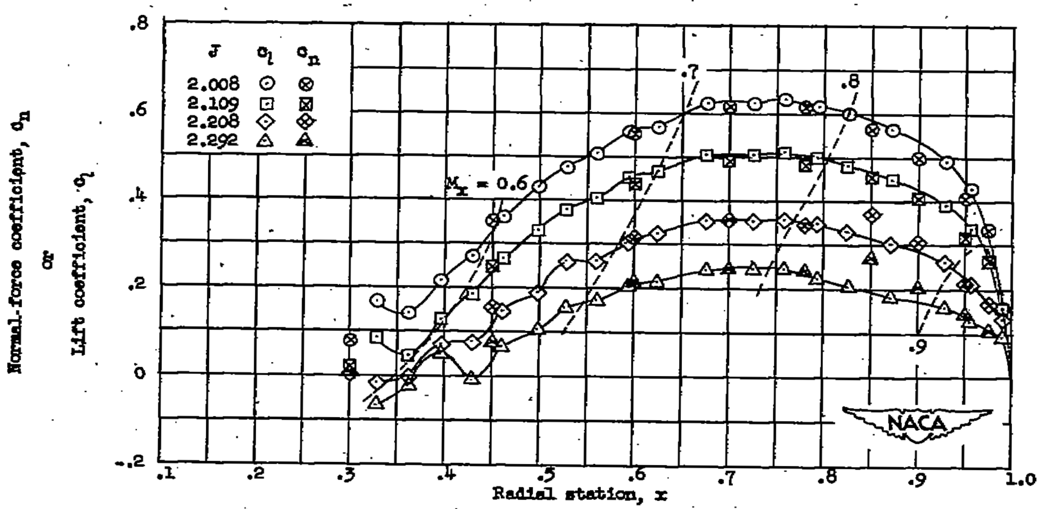



(b) 1350 rpm; $\beta_{0.75R} = 45^{\circ}$.

Figure 3.- Comparison of lift coefficient $c_l$ from wake survey with pressure distribution normal-force coefficient $c_n$.

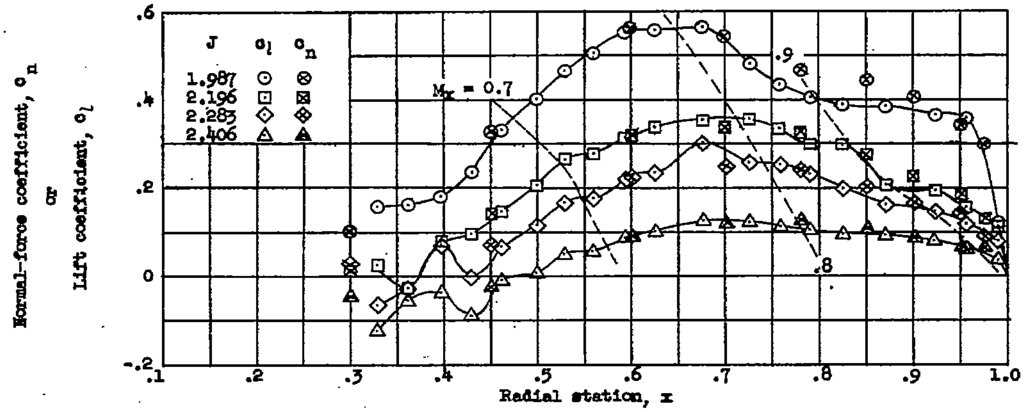



(c) 1500 rpm; $\beta_{0.75R} = 45^{\circ}$.

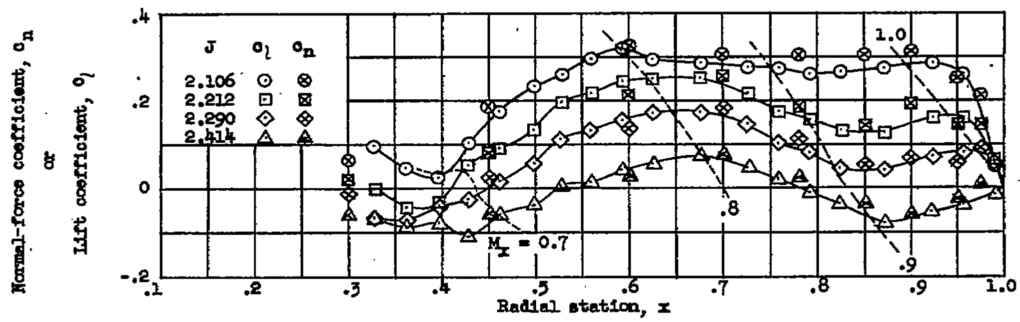



(d) 1600 rpm; $\beta_{0.75R} = 45^{\circ}$.

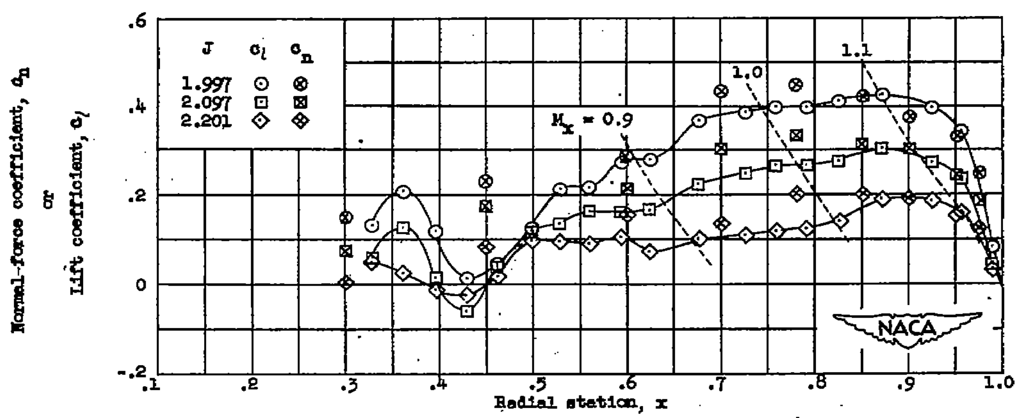
Figure 3.- Continued.




(e) $M_o = 0.56$; $\beta_{0.75R} = 45^o$.




(f) $M_o = 0.60$; $\beta_{0.75R} = 45^o$.




(g) $M_o = 0.65$; $\beta_{0.75R} = 45^o$.

Figure 3.- Concluded.

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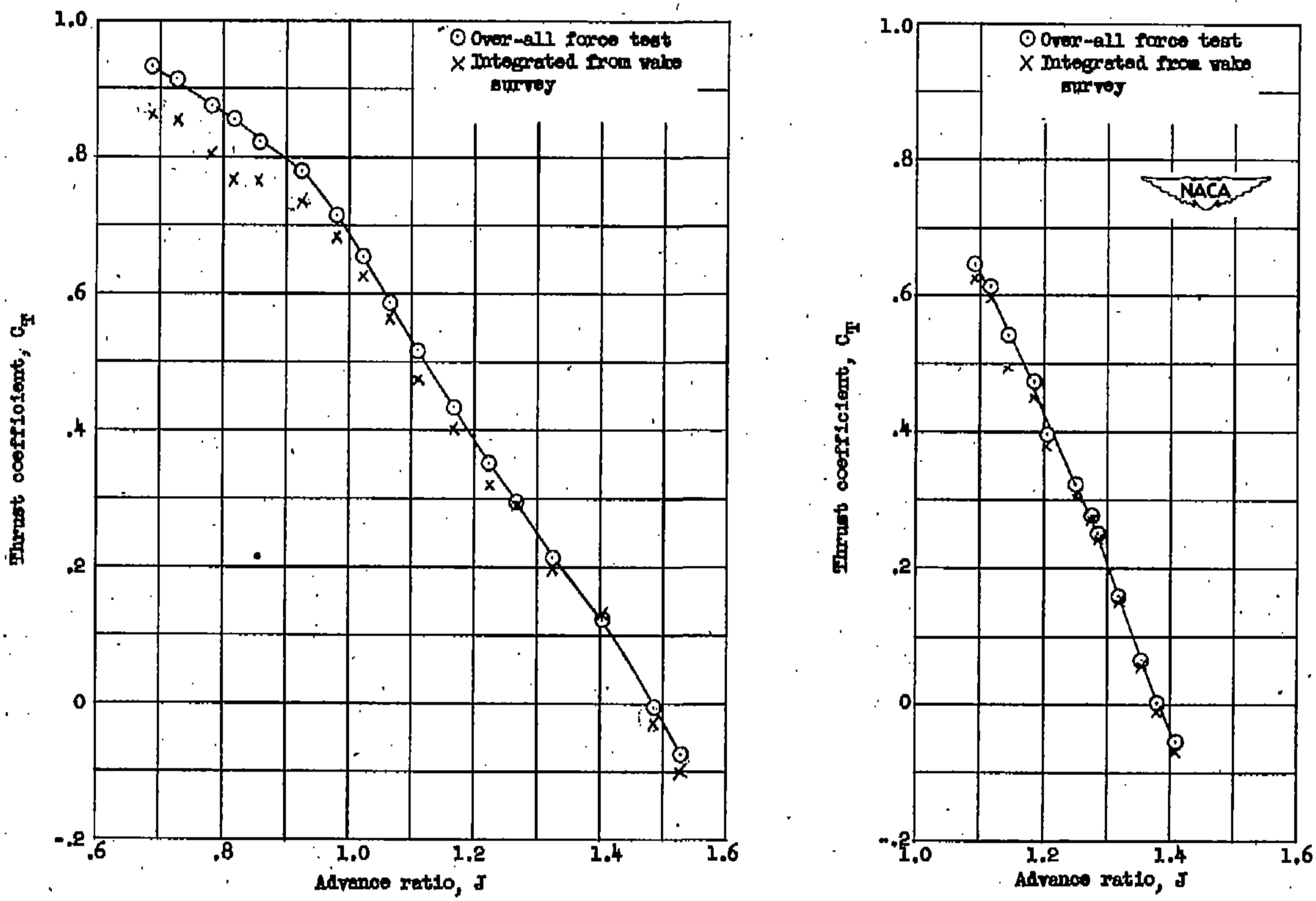
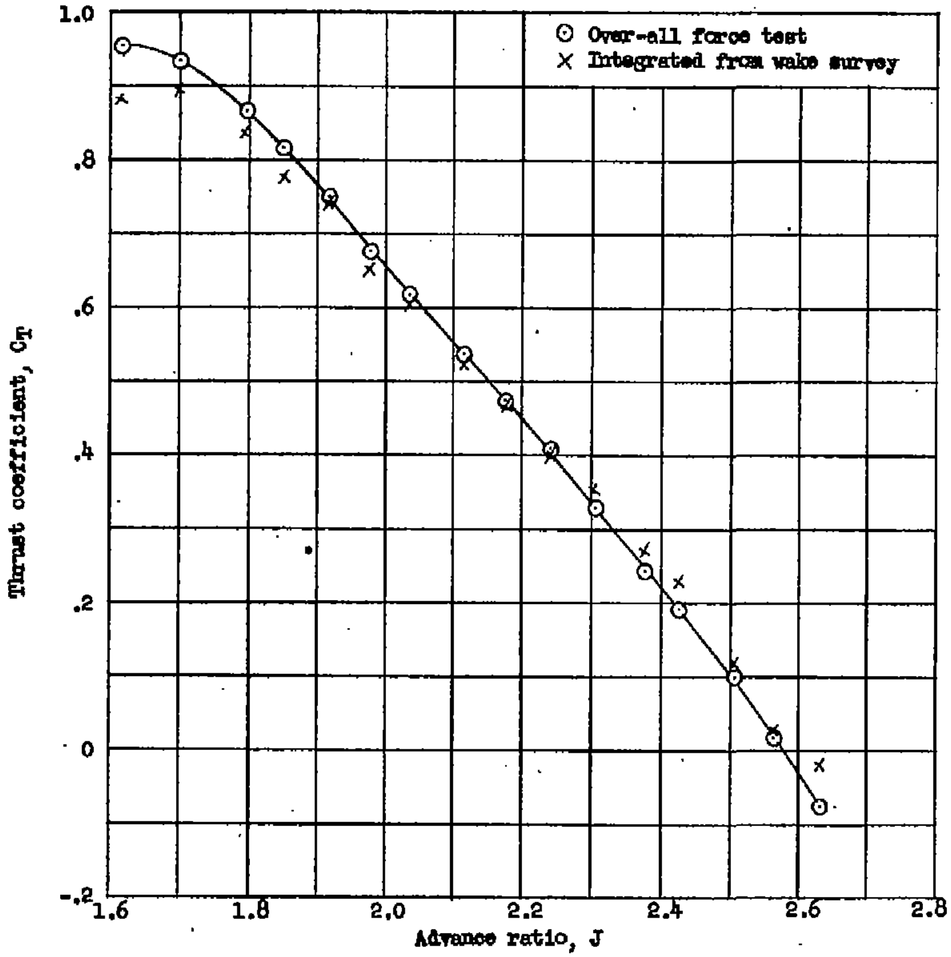


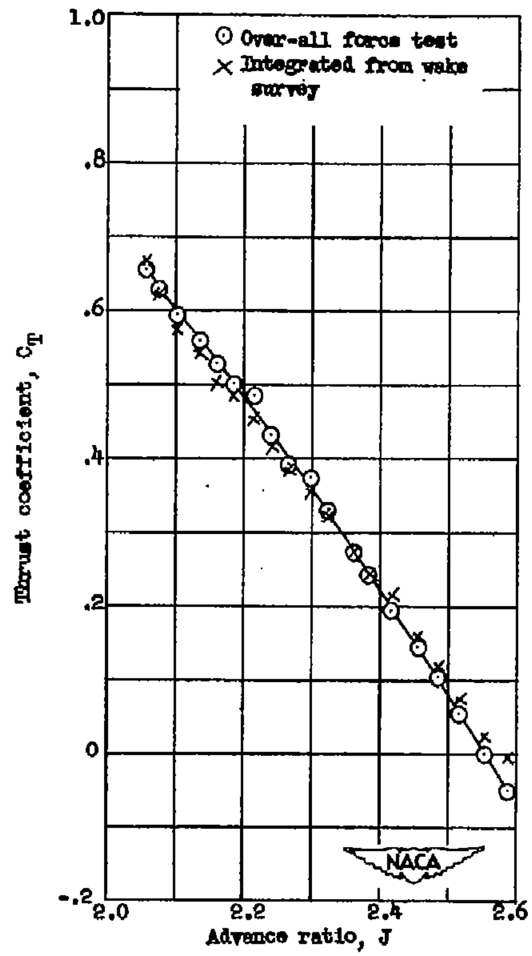

(a) 1140 rpm; $\beta_{0.75R} = 30^\circ$.

(b) 2160 rpm; $\beta_{0.75R} = 30^\circ$.

Figure 4.- Comparison of thrust coefficient from wake survey with that from over-all force test.




(c) 1140 rpm; $\beta_{0.75R} = 45^{\circ}$.




(d) $M_o = 0.56$; $\beta_{0.75R} = 45^{\circ}$.

Figure 4.- Continued.

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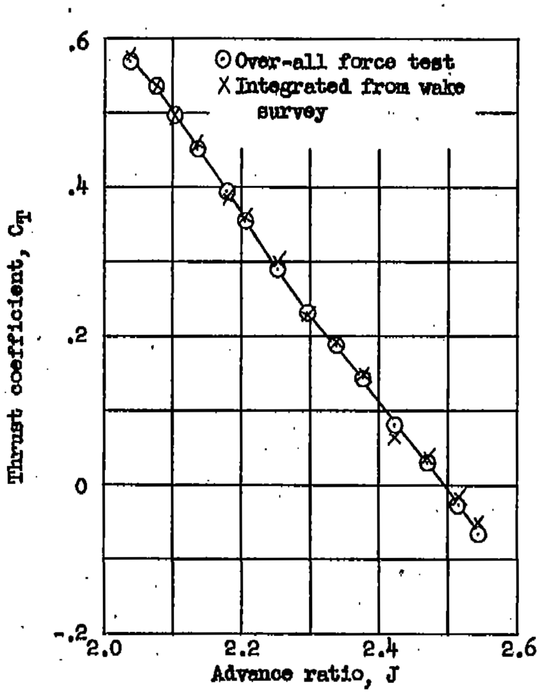
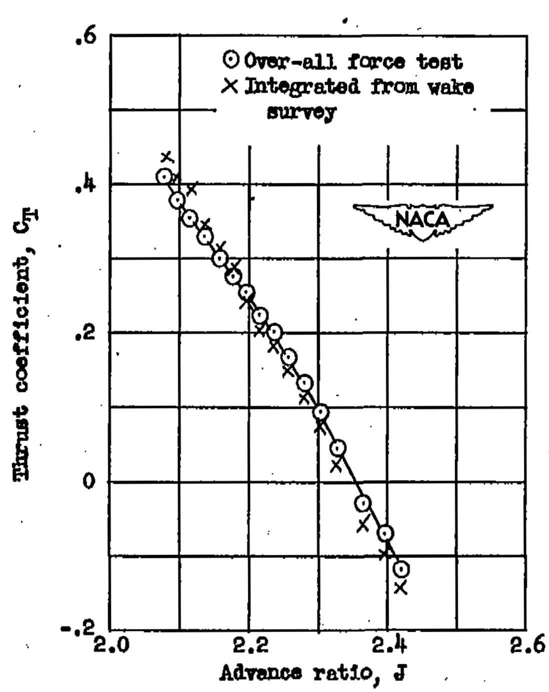



(e) $M_o = 0.60$; $\beta_{0.75R} = 45^{\circ}$.




(f) $M_o = 0.65$; $\beta_{0.75R} = 45^{\circ}$.

Figure 4.- Concluded.

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